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ABSTRACT

This qualitative assessment of tenth-grade students' (n=17) problem solving focused on the nature of students' thinking, their problem-solving strategies and heuristics, the mathematical approaches they selected, and the ways they monitored their own progress. The problem, presented orally and with photographs in a task-based interview, involved a sloping ramp connecting a fixed dock with a floating dock. The most direct method of solving the problem was to apply the Pythagorean theorem. There were two respects in which the context of this activity seemed to influence performance: (1) Students had difficulty translating from the given problem situation to a mathematical representation, and (2) because the problem involved right triangles, which the students had recently been studying, many students initially attempted to attack the problem using trigonometric tools, which was either inefficient or confusing. The discussion of results includes consideration of persistence in seeking a solution and cognitive monitoring of solution processes. Contains 11 references. (MKR)

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Research Symposium:

Roles for Context in Mathematics Education

Using the Pythagorean Theorem in a Contextualized Problem

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ABSTRACT

This paper draws its data from a qualitative assessment of Grade 10 students' problem solving that focused on the nature of students' thinking, their problem-solving strategies and heuristics, the mathematical approaches that they selected, and the ways they monitored their progress. The problem, presented orally and with photographs in a task-based interview, involved a sloping ramp connecting a fixed dock with a floating dock that moves up and down with the tide. The most direct method of solving the problem is to apply the Pythagorean theorem to two right triangles each having the ramp as hypotenuse, one representing the situation at high tide, the other the situation at low tide. The difficulties students encountered in solving the problem seemed to be metacognitive, having to do mainly with identifying and representing the salient features of the situation in diagrams, and also deciding what mathematical processes to apply. Although the problem can be solved most directly by applying the Pythagorean theorem, many students initially tried using trigonometry.

There were two respects in which the context of this activity seemed to influence students' performance. First, it was the task of decontextualizing the given elements of the problem (the vertical and horizontal measurements and the length of a sloping ramp), rather than the tasks of operating on those elements as parts of diagrams representing the situation, that consumed most of the students' time and caused many of their difficulties. For these students the abstracted, decontextualized mathematical representations they eventually produced seemed easier to deal with than the contextualized problem elements. Second, since the problem involved right triangles, and since their current mathematics course had just recently begun introducing them to definitions of trigonometric ratios in right triangles, many students initially attempted to attack the problem using trigonometric tools. This resulted either in their solving the problem correctly but inefficiently, or more often, in their becoming lost in the details of trying to use the trigonometric ratios. Even though the problem was not presented as part of their regular mathematics course, some students' work on it was strongly influenced by their mathematics-course-context, and in many cases their cognitive monitoring function did not over-rule that influence.

Using the Pythagorean Theorem in a Contextualized Problem

In countries all around the world, mathematics curricula for the lower levels of secondary education include the Pythagorean theorem, usually with intended learning outcomes stating that students should be able to apply the Pythagorean relationship both to determine whether a given triangle is a right triangle and to calculate the length of a third side of a right triangle given the other two sides. Current curricula (e.g. British Columbia Ministry of Education, 1988) often emphasize practical problem solving situations (e.g., how far up a wall does a ladder reach?) and may indicate that a calculator should be used as appropriate. The results discussed in this paper are part of a project that was conducted in conjunction with a large-scale assessment (Robitaille, 1991; Robitaille, Schroeder, & Nicol, 1991) that used achievement test items in multiple-choice and constructed-response format. The goals of the small-scale study were (1) to develop a series of non-routine problems that would be challenging yet accessible, that would demand planning and reflection, that would permit a number of different methods of solution, and that would embody a variety of familiar mathematical processes and operations, but not in obvious, routine ways; and (2) to use those problems in interviews with individuals and pairs of students to describe and assess the mathematical processes and operations they apply and the problem solving plans and strategies they adopt. One of the seven problems developed for Grade 10 students, referred to as the "dock" problem, had content related to the Pythagorean theorem.

The discussion in this paper focuses on influences of context on the students' performance. Additional details about the students' performance and discussion of other aspects of their problem solving may be found in other reports (Schroeder, 1992a, 1992b, 1993). Implications of the findings for practice and for further research are also suggested.

Interview Task. The task was presented to students orally by an interviewer who explained the problem situation with the help of photographs. Students were told that the waterfront restaurant shown in Figure 1 has a dock, one part of which rises and falls with the tide. Access from the fixed part of the dock to the floating part is by means of a ramp which is relatively steep at low tide (Figure 1a), but less steep at high tide (Figure 1b). Currently, the lower end of the ramp rests on the floating dock (Figure 2a). As the floating dock rises and falls, the end of the ramp scrapes across it causing scratches in the surface. In order to prevent this damage, the owners of the dock plan to mount wheels on the lower end of the ramp and tracks on the floating dock for the wheels to run in – an arrangement similar to the one shown in Figure 2b, which shows another dock nearby. The problem is to determine how long the track should be. The data, provided on a separate sheet, are that (1) the ramp is 18 m long, (2) when the tide is at its highest, the floating dock is 1 m below the fixed dock, and (3) when the tide is at its lowest, the floating dock is 6 m below the fixed dock.

Figure 1 and Figure 2 here.

It was anticipated that students would approach the problem by drawing one or more diagrams and by recognizing the importance of two right triangles, both having the 18 m long ramp as the hypotenuse, one 1 m on its vertical side, the other 6 m tall. The difference between the lengths of the horizontal sides of these two triangles is the required length of the track. Thus the simplest method of solving the problem is to use the Pythagorean theorem to determine the lengths of the two unknown horizontal sides and

then subtract. It was anticipated that students might use other means of solving the problem such as constructing a scale drawing or applying trigonometric ratios to calculate the unknown lengths.

Procedures. Seventeen volunteers, 11 females and 6 males, took part in interviews based on this problem; thirteen were interviewed individually, the remaining four in two same-sex pairs. The students were provided with the photographs, the summary of the data, a scientific calculator, tables of square roots and trigonometric functions, squared paper and plain paper, and a geometry set consisting of a ruler, a protractor, a set square, and a compass. In the introduction to the interviews students were informed that the activity was not a test and that their performance would not affect their standing in their mathematics course in school. They were told that the researcher conducting the interviews was more interested in how they went about solving the problem than in whether they got the correct answer, and that they could ask questions of the interviewer at any time. Students were urged to think aloud as they solved the problem so that the interviewer could understand how they worked on it, and they were told that the interviewer might ask them questions for clarification, and would give them hints or help if they wished.

As the students worked on the problem, the interviewer observed them closely and made field notes. At the conclusion of each interview the interviewer completed a record sheet designed to be a convenient and standardized way of summarizing and reporting students' work on the problem. Sections of the record sheet headed "Understanding," "Strategy Selection," and "Monitoring" reflect Polya's (1945) description of the phases of problem solving and the importance of metacognitive activity in problem solving; they list a number of anticipated features of students' work which can be checked off as appropriate, and provide spaces in which to describe students' work. In a final section headed "Overview" the interviewer is to indicate whether the student(s) solved the problem essentially on their own, or solved the problem with needed help from the interviewer, or did not solve the problem even with help from the interviewer. The amounts of time spent reaching a solution, extending the problem or looking back, and using different approaches are also to be noted.

Results. In the discussion which follows, the unit of analysis is the interview, of which there were 15. The overall results showed that in five interviews (33%) the students solved the problem on their own, and that in ten interviews (67%) the students solved the problem with help from the interviewer; in none of the interviews did the students fail to solve the problem. The total length of time spent in each interview varied from 22 to 50 minutes with a median of 38 and a mean of 39, and the time taken to reach a solution ranged from 9 to 50 minutes with a median of 18 and a mean of 22. Interviews in which the students solved the problem on their own tended to be somewhat shorter overall and noticeably shorter in time to solution. The median time to solution for students who solved the problem on their own was 10 minutes, as opposed to 24 minutes for students who received help. Although these gross measures give a sense of the extent of the interviews and an idea of how well the students performed, they were not the focus of the analysis; the qualities of students' work was the main concern.

It was anticipated that students would draw diagrams both as a means of understanding the problem and to facilitate their work on it. Although all students eventually solved the problem using diagrams that included two right triangles, there were wide variations in their initial drawings, some of which are shown in Figure 3. In these early diagrams different features of the problem are prominent, and in some of them critical features of the problem are misrepresented. For example, in two interviews students first represented the situation as in Figure 3a with three parallel lines. Two students initially

drew diagrams similar to the one in Figure 3b with the track in the plane of the ramp rather than the plane of the floating dock. The student who drew the diagram in Figure 3c focussed on the rotation of the ramp about its upper end. As she examined the photos she rotated her pencil holding the upper end stationary just above the paper and allowing the point to trace out an arc on the paper.

Figure 3 here.

The five students who solved the problem on their own quickly produced appropriate diagrams in which the two needed right triangles were prominent. In four cases the triangles were drawn in two separate figures; in one case they were overlapping as in Figure 3f. In five of the ten interviews where the problem was solved with help from the interviewer, students produced diagrams on their own which they used to make progress toward a solution; the help they received was unrelated to representing the problem in a diagram.

In the remaining five interviews students received help that was related to representing the situation in diagrams and identifying the relevant parts of diagrams they had drawn. In one case, after the student had spent some time studying the photographs and appeared to be stuck, the interviewer suggested that it might help to draw a diagram. In two cases the students had drawn appropriate overlapping diagrams (similar to Figure 3f), but after several minutes had not made progress using them. In one of these cases the interviewer asked, "Are there any triangles in the diagram that you could use?" and in the other he said, "Would it help to draw two separate diagrams?" to which the student immediately replied, "You mean one for high tide and one for low tide?" Each of these hints led immediately to progress toward a solution. The difficulties experienced in the remaining two interviews seemed to be related to misconceptions regarding the names and relative positions of the fixed dock, the ramp, the floating dock, and the linkages between them. These students produced the initial diagrams shown in Figures 3g and 3h. Their difficulties were resolved in question and answer exchanges between the interviewer and the students which focussed the terms, the photos, the data, and the students' diagrams.

It was anticipated that students would use the Pythagorean theorem to find the horizontal dimensions of the two right triangles formed by the floating dock, the vertical, and the ramp, and in all 15 interviews students sooner or later obtained a solution in this way. In 11 of the interviews (73%) students used the Pythagorean theorem without being given a hint that they should do so, and without receiving any help in applying it to the figures they had drawn. The helps and hints given in the remaining four interviews (27%) ranged from the fairly oblique, "Is there any way you could relate the side you want to the sides you know?" in one case, to the quite direct, "Would Pythagoras's theorem help?" in another. A third student wondered aloud whether the angle was a 90° angle, and was asked by the interviewer, "What if it was 90° and what if it wasn't?" to which she replied, "If it was, I could use Pythagoras." In another interview the two students had spent more than 40 minutes trying various approaches without success, when one of them asked, "What's the square root table for?" The students decided on their own that it could be a hint to use Pythagoras, and before long they reached a solution by this method.

All of the students seemed to be quite familiar with the Pythagorean theorem, although one student referred to his method as "using a theory," and another referred to it as "Mr. So-and-so's method," presumably because that teacher had taught or reviewed it. In cases where the students received hints related to the Pythagorean theorem, the hints were mostly vague questions (e.g., What do you know about this triangle? ... What can

you find out?) rather than direct hints, and they concerned whether to use the Pythagorean theorem, not how to use it. There were no instances in which students made errors using the Pythagorean theorem that they did not detect and correct by themselves (e.g. failing to square or take the square root, adding the squares rather than subtracting them, making computational errors, etc.).

Application of the Pythagorean theorem was not, however, the first approach adopted in all the interviews; in seven interviews (47%) students began by using or proposing to use trigonometry. One student produced his first solution using trigonometry, but most of the students abandoned this approach either because they ran into difficulties with it or because they noticed that applying the Pythagorean theorem would be simpler. In all cases where there was time available, students who had found the solution were asked if they could solve the problem in another way. In six interviews (40%) students solved the problem using trigonometry. In two of these interviews the students produced a trigonometric solution without help from the interviewer, but assistance of various types was given in the other four. Two of the students commented that they were just starting to learn trigonometry in their mathematics class; they thought trigonometry could be used, but they weren't sure they could do so successfully. The fact that the students had only recently begun studying trigonometry probably accounts for the large number of students who thought of using it and for the difficulties they encountered in doing so.

Before the interviews were conducted it was anticipated that some students might use scale drawing as a means of solving the problem, and for that reason a geometry set was provided. None of the students proposed solving the problem with a scale drawing, and two students thought it would not be possible when the interviewer suggested it.

Discussion. One of the most remarkable findings of this study was the amount of time that the students spent working on the problem. By comparison with multiple-choice test items, which students are expected to answer at the rate of about one per minute (Taylor, 1991), or constructed-response items, which take on the order of five minutes (Szetela, 1991), this task was quite time consuming. Almost all students spent more than half an hour on the problem, and the average time to solution was on the order of 20 minutes. The amount of time that the students spent not only gives a measure of their perseverance with the task and their willingness to reflect on and extend their work; it also suggests that many students found this problem challenging and difficult. Features of the problem that seem to have contributed to the difficulties are discussed in the following paragraphs

In five of the interviews (33%) students readily drew the two right triangles that are key to solving the problem, and in an additional five interviews (33%) students drew appropriate diagrams without being given specific help, but they spent a great deal longer producing their diagrams, and in some cases did so for only one of the two conditions. Representing the problem situation in useful diagrams was a major source of difficulty in the remaining five interviews (33%). One reason for the difficulties with diagrams may be that only one side of the two needed triangles, the sloping ramp, is concrete; the other two sides must be constructed or imagined. One of them is a vertical line extending downward, from the point where the upper end of the ramp meets the fixed dock, to the plane of the surface of the floating dock. In the photographs one cannot "see" this line, since it does not correspond to any structural element of the dock system; it is an imaginary line through empty space. Similarly, the horizontal side is a line in the plane of the floating dock that starts on the surface of the floating dock where the scratches are, and that extends over the surface of the water (a fraction of a metre above it) until it intersects with the previously described vertical. In two of the photographs (Figures 1a and 1b) these imagined lines can be "seen" to be perpendicular, but the horizontal line is obscured by objects in the

foreground. The photograph shown in Figure 2a gives a close-up of the area in which these lines need to be constructed, but because of the photograph's perspective they might not appear to be at right angles.

The students who initially made diagrams like the one shown in Figure 3a drew horizontal lines representing the planes of the fixed dock and the floating dock at high tide and at low tide, but the numeric values written on these diagrams are not measurements along these lines. Before these numeric values can be put to use, vertical lines must be added to the diagram and the measurements must be appropriately related to them. A key understanding required in order to construct an adequate representation of the problem is that the given distances below the fixed dock are measured along vertical lines, and that these verticals may be placed wherever it is convenient or necessary, even through empty space. Horizontal line segments must also be added to all of the diagrams (except that of Figure 3f) to represent the horizontal components of the ramp at high tide and low tide. In the diagram shown in Figure 3e it is probably significant that the relative positions of the wheel on the end of the ramp at high tide (indicated by the "H") and at low tide (indicated by the "L") are correctly identified. The diagram shown in Figure 3d, on the other hand, erroneously suggests that the end of the ramp will be farther out at low tide than at high tide.

The student who drew the diagram shown in Figure 3c saw the sloping ramp not as the hypotenuse of a right triangle but as the radius of an arc, the path travelled by the lower end of the ramp where the wheel is to be placed. Her diagram also contains two vertical lines which could be marked to correspond to the given distances below the fixed dock, but there are no horizontal lines corresponding to the fixed dock, or the track, or the horizontal components of the ramp at high and low tides. Without some such horizontal lines appropriately identified with elements of the problem situation, this diagram is not particularly helpful. If the student had added the dashed horizontal line shown in Figure 4 and had extended the two original vertical lines to meet it, she would have had a complete and useful diagram in which the lengths of the track and the horizontal components of the ramp can be found in the plane of the fixed dock rather than in the plane of the floating dock as discussed previously. However, it is probably unlikely to expect this, since the two triangles would then be "upside down," and the needed horizontals would not be found in the plane of the floating dock, where one expects them, but as projections onto the plane of the fixed dock. In only one interview (7%) was a diagram produced with two separate triangles "upside down"; overlapping triangles as shown in Figure 3f were drawn in three of the fifteen interviews (20%).

Figure 4 here.

The various diagrams included in Figure 3, and the foregoing comments about them and other possible diagrams, illustrate the complexities and difficulties inherent in this problem.

As was noted above, students did not appear to have difficulties related to *how* to use the Pythagorean theorem, although some of their difficulties seemed to pertain to *whether* to use it. This finding contrasts sharply with a finding of the large-scale assessment with respect to Mathematics 10 students' performance on five multiple-choice items involving the Pythagorean theorem. (See Figures 5a to 5e.) An interpretation panel called the results on these five items "disappointing" and concluded from them that "students did not have an understanding of the Pythagorean theorem" (Taylor, 1991, p. 158). It is clear that the students who were given the dock problem (and who may not be

representative of the whole population tested in the large-scale assessment) did *not* lack understanding of the Pythagorean theorem nor related computational skills. Their difficulties had to do with general problem-solving skills and metacognitive abilities, in particular representing the problem in diagrams, identifying relevant features, and planning what to do. The interview task was obviously challenging because it demanded a process that may be described as decontextualization, as mathematization, or as mathematical representation. The students gave the impression that their school mathematics courses had not provided many opportunities to practice the such processes. Although there are no data to prove this, it seems likely that their previous experiences applying the Pythagorean theorem had usually been in situations where a diagram was either supplied or readily drawn (as in the case of the item shown in Figure 5e, for example).

Figure 5 here.

A different aspect of the context in which the interview task was given, and an important one for its influence on students' performance, is the content of the students' school mathematics course. Around the time of the interviews, students were being introduced in Mathematics 10 to trigonometric ratios and their use in solving right triangles. Having recently seen the definitions of the ratios and their mnemonics in a form like the one shown in Figure 5, it is perhaps not surprising that students would attempt to use the opposite side and hypotenuse to find the angle of elevation of the ramp, and then the cosine of that angle and the hypotenuse to find the adjacent horizontal side of the triangle. In fact, in nearly half of the interviews (47%) that was the students' initial approach.

Figure 6 here.

Finally, the context of the interview in which the problem was posed must be considered. Students were provided tools that might be useful in solving the problem, including a scientific calculator, tables of square roots and trigonometric functions, squared paper and plain paper, and a geometry set. In one interview, the one of the pair of students wondered out loud whether the square root table might be a hint to use Pythagoras. Similarly, some students might have been influenced by the presence trigonometric tables to adopt a trigonometric approach, although there is no direct evidence of this. The provision of graph paper and a ruler, however, did not lead any of the students to attempt to solve the problem using a scale drawing.

Concluding Remarks. In the contemporary mathematics reform movement there is a great deal of interest in applications of mathematics, real-world problem solving, and mathematical connections with other content areas (National Council of Teachers of Mathematics, 1989). Although there is a great deal of professional literature arguing the need for these components in the mathematics curriculum and suggesting potential topics for classroom activities, there is little in the way of pedagogical literature documenting the thinking and actions of students and teachers actually involved in such activities, and apparently even less research focussed on teaching and learning about representations and problem solving of the type documented in this paper (e.g., Treilibs, 1979). A research program focussing on influences of context on students' performance in these areas is clearly demanded.

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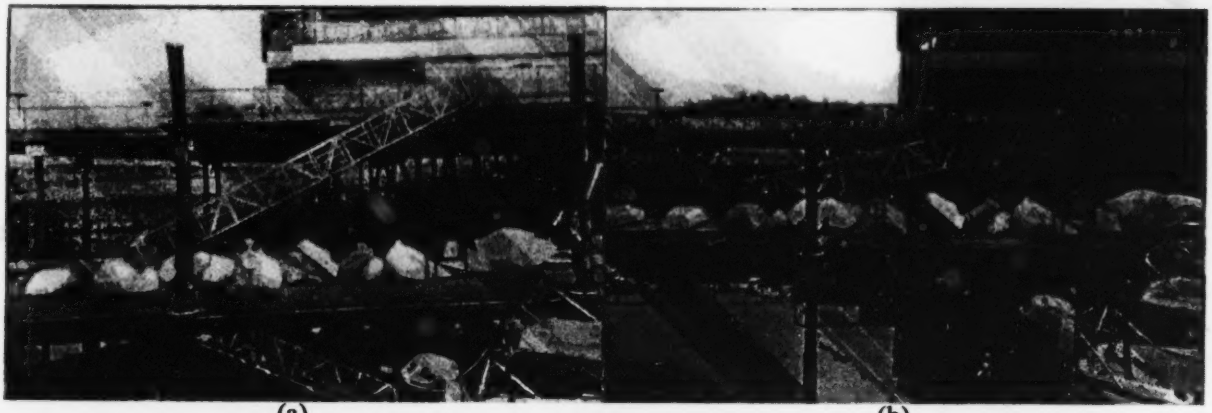

(a) (b)

Figure 1: A waterfront restaurant and its dock at low tide and at high tide.

(a) (b)

Figure 2: The lower end of the ramp at present and as proposed.

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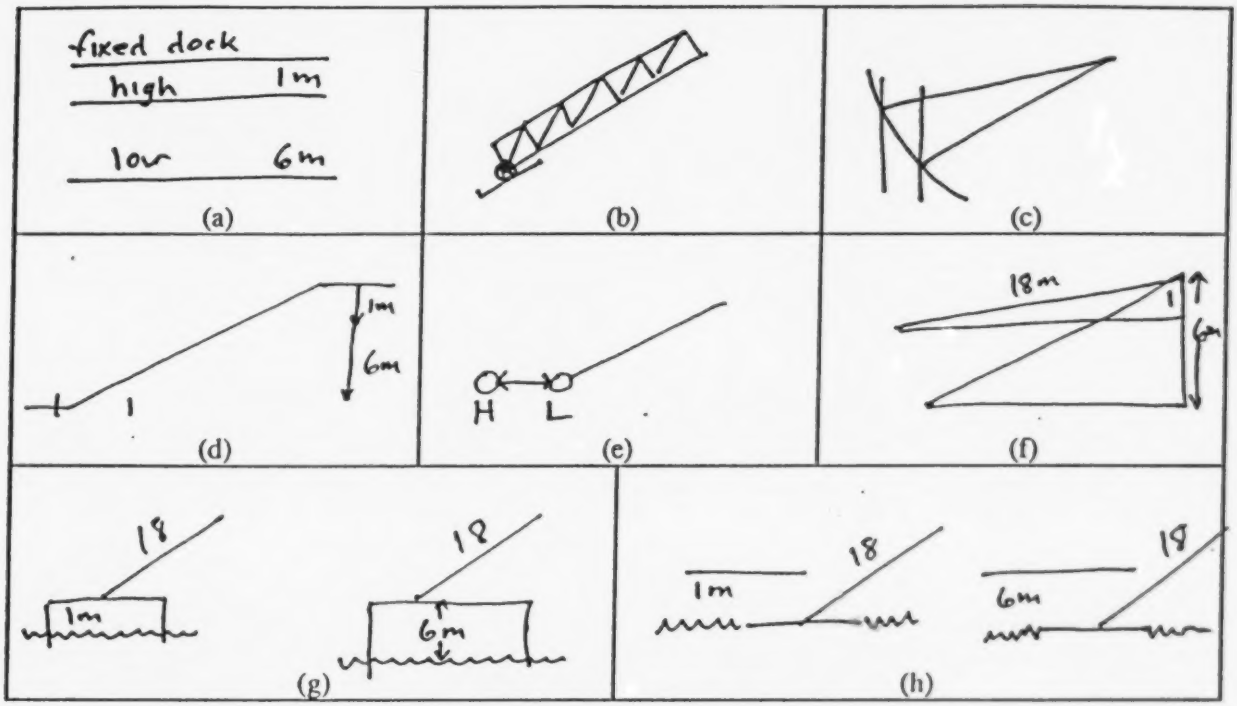


Figure 3: Initial diagrams drawn by students.

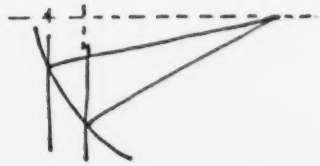
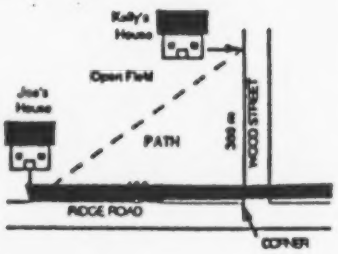


Figure 4: A modification of the diagram in Figure 3c

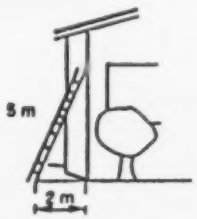
3.2 A 11. When Joe walks from his house to Kelly's house, he follows the path through the open field. How far does he walk?

16 A) 450 m
56 B) 500 m
10 C) 550 m
10 D) 600 m
8 E) I don't know.



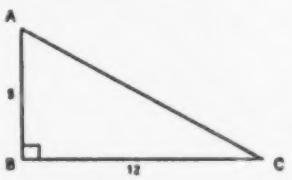
3.3 A 12. Joe, the window cleaner, uses a ladder that is 5 m long. When he places the foot of the ladder 2 m from the wall of the house, how high up the wall does the ladder reach?

17 A) $\sqrt{10}$ m
35 B) $\sqrt{29}$ m
41 C) $\sqrt{21}$ m
12 D) 7 m
14 E) I don't know.



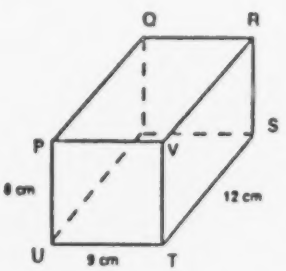
3.3 C 12. $\triangle ABC$ is a right triangle. Determine the length of side AC.

53 A) 13
22 B) 17
5 C) 49
6 D) 169
8 E) I don't know.



3.3 C 60. In the rectangular solid below, the length of the diagonal PS is

13 A) 21 cm
10 B) 17 cm
17 C) 20 cm
8 D) $\sqrt{200}$ cm
27 E) I don't know.



3.4 B 50. An ocean liner travels 10 km north and then 8 km east. How far is the ship from its starting point?

52 A) 11.7 km
25 B) 18 km
11 C) 8 km
6 D) 4 km
5 E) I don't know.

Figure 5: Multiple-choice items related to the Pythagorean theorem.

$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$	S. O. H.
$\text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$	C. A. H.
$\text{tangent} = \frac{\text{opposite}}{\text{adjacent}}$	T. O. A.

Figure 6: Presentation of trigonometric definitions